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A Method of Curvature Matching for Two-Dimensional Flexible Plate Wind-Tunnel Nozzles

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Nomenclature

A = area under the aerodynamic K-S plot
E = Young's modulus of elasticity
I = moment of inertia of the plate cross section
K = curvature
m = slope of the linear K-S plot
R = total load applied on a jack
S = distance along the plate
$[A]$ = area matrix
$[C]$ = coefficient matrix
$[K]$ = unknown curvature matrix

Subscripts

i = ith station on the plate contour
av = average

Introduction

In order to simulate the contour of a two-dimensional wind-tunnel nozzle for continuously varying test section Mach number, it is usual to employ two flexible plates loaded transversely by a finite number of jacks along its length. It is impossible to exactly simulate the aerodynamic contour by the device previously mentioned. If the jack extensions are adjusted to make the ordinates of the plate at a finite number of points equal to those of the aerodynamic contour at these points, the slope and the curvature distributions of the plate deviate from those of the contour, resulting in nonuniformity in the Mach number distribution in the test section. For obtaining a uniform Mach number distribution, it is necessary to simulate the slope distribution exactly.

It may be noted that the exact simulation of the slope distribution throughout the plate length is impossible. For practical purposes, it is sufficient to ensure identical slopes at the finite number of points. This can be done by matching the curvature distribution of the plate with that of the aerodynamic contour.

This Note describes a method of curvature matching adopted for the nozzle contours of the 4 ft × 4 ft trisonic wind tunnel of National Aeronautical Laboratory, India (NAL). As a consequence of matching, a set of over-determined, simultaneous equations is obtained. A least-square solution of these equations gives the curvature distribution which ensures minimum slope error.

The analysis for the derivation of this set of equations follows.

Analysis

The arrangement of the jacks used for loading the plate is shown in Fig. 1 (upper). There are 17 jacks; seven of them have single attachment with the plate and the remaining ten are whiffle-tree jacks having two attachments.

The curvature distributions (the $K - S$ plot) of the aerodynamic contour and the plate are shown in Fig. 1 (lower). It may be noted that the curvature of the plate varies linearly between any two-successive jacks. The area shown shaded in this figure represents the slope error between two jacks. The problem of curvature matching consists in selecting the curvatures at the 27 attachments in such a way that the slope-error between any two-successive jacks is zero. This condition gives rise to the following 26 equations connecting $K_1, K_2 \ldots K_{27}$.

$$K_i + K_{i+1} = 2A_{i,i+1}/(S_{i+1} - S_i);\ i = 1,2,\ldots 26 \quad (1)$$

Here $A_{i,i+1}$ is the area under the K-S plot for the aerodynamic contour between ith and $(i + 1)$th jack and is known from the contour design.[1,2,3,4,5]

When the whiffle-tree jacks are used, there are additional constraints on $K_1, K_2 \ldots K_{27}$. These constraints arise as explained below.

Around a typical whiffle-tree jack, we have a curvature distribution as shown in Fig. 2. The following relations hold for the shear force on the plate

$$EIm_j + R_1 = EIm_{(j+1)} \quad (2)$$

$$EIm_{j+1} + R_2 = EIm_{(j+2)} \quad (3)$$

where

$$R_1 + R_2 = R \text{ (Fig. 2)}$$

From these equations we have

$$2EIm_{(j+1)} = EIm_{(j+2)} + EIm_j + (R_1 - R_2) \quad (4)$$

If the linkages for the whiffle-tree jacks are such that $R_1 = R_2 = R/2$, we get a relation connecting the slopes m_j only

$$m_{j+1} = (m_{j+2} + m_j)/2 \quad (5)$$

There are 10 equations of Eq. (5) type.

Equations (1) together with Eq. (5) form a set of simultaneous equations in the unknown K_i. The number of equations is 36.

In a matrix notation these equations can be written as

$$[C]\cdot[K] = [A]$$

A least-square analysis gives

$$[K] = [M]^{-1}\cdot[C]^T\cdot[A]$$

where

$$[M] = [C]^T\cdot[C]$$

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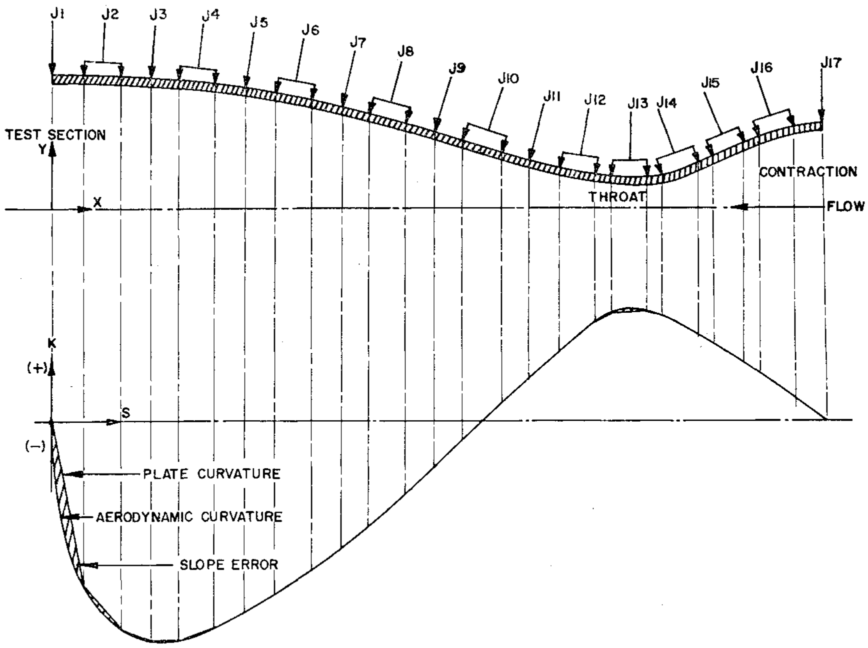

Fig. 1 Jack arrangements and the curvature distributions.

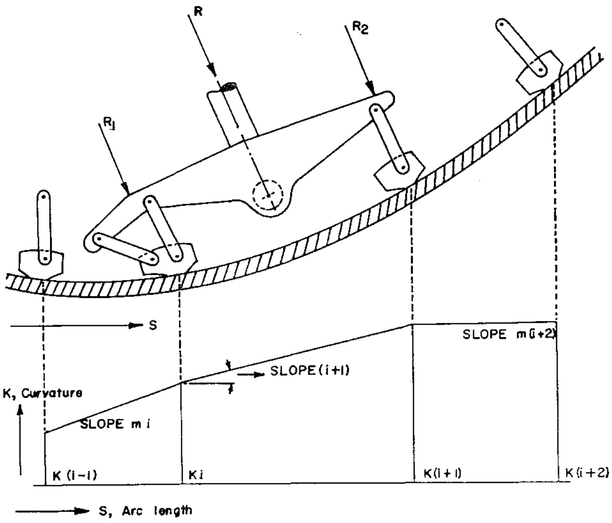

Fig. 2 A typical whiffle-tree jack and the corresponding plate curvature.

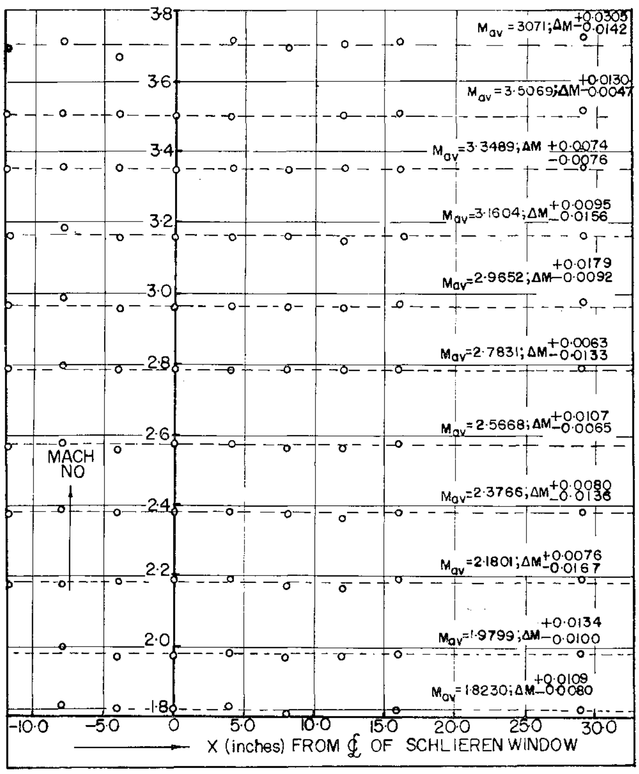

Fig. 3 Measured axial Mach number distribution in the test section.

It may be noted that $[C]$ is constant for a given tunnel; only $[A]$ varies with the test section Mach number.

The final curvatures $[K] = K_1, K_2, \ldots K_{27}$ will not, however, satisfy the given equations exactly. At the end of matching a nonzero slope error is still present. These errors are found to be within the limits specified. With this curvature distribution, the coordinates are calculated for setting the flexible plates of the wind tunnel. The resulting Mach number distribution for a few test-section Mach numbers is shown in Fig. 3.

Conclusion

For the tunnel under consideration the limit on the variation of Mach number in the test section is 0.5%. The contours obtained by the previously mentioned analysis are found to give Mach number distribution in the test section within this limit.

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